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STABILITY OF A PLASTIC CYLINDRICAL BAR IN TORSION

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The stability of a cylindrical bar twisted by soft and hard loading schemes is studied. The material is deformed plastically without continual failure. It is assumed that the material becomes physically unstable after hardening (softening stage). Two new criteria are used to determine the instability moment and strain localization.

Introduction. The instability and strain localization constrain the deformation of a solid body and lead to its failure. These complex phenomena are not clearly understood.

As a rule, physical instability of a material is ignored in studying instability of solids [1, 2]. Physical instability is usually taken into account by considering the necessary conditions of strain localization for the uniform stress–strain state regardless of the body geometry, loading conditions, and the deformation region of the material in the stages of elastic deformation, hardening, and softening (physical instability) [3].

In the present study, the stability problem of a twisted cylindrical bar is considered with allowance for all factors affecting stability and strain localization (length and radius of the cylinder, loading conditions, physical instability of the material, and stress–strain state) under the assumption of ideal plastic properties of the material, i.e., the damage (continual failure) is ignored. Two criteria are proposed to determine the instability moment and strain localization.

1. Properties of the Material. We consider a twisted bar of length l and cross-sectional radius R. The deformation is caused by the torque M (soft loading) or the twist angle ψ (hard loading). We assume that the properties of the material are characterized by a strain diagram in the coordinates τ (shear stress) and γ (shear strain), which consists of ascending and descending branches [4]. The elastic behavior of the material is characterized by the shear modulus G; material hardening (ascending branch $\gamma^{\rm yield} < \gamma \leqslant \gamma^{\rm t}$) following the linear sector of elasticity and softening (descending branch $\gamma^{\rm t} < \gamma \leqslant \gamma^z$) are characterized by the tangent (instantaneous) shear modulus $G^{\rm p} = d\tau/d\gamma$. Here $\gamma^{\rm yield}$, $\gamma^{\rm t}$, and γ^z are the shear strains corresponding to the yield point $\tau^{\rm yield}$, ultimate strength $\tau^{\rm t}$, and failure, respectively.

Depending on the processes occurring in the material upon deformation, the following three variants of unloading are possible: 1) no residual strains occur, and unloading is characterized by the secant modulus $G^s = \tau/\gamma$; 2) residual strains occur, and the unloading modulus is equal to G; 3) residual strains occur, and the unloading modulus is equal to G^u ($G^s < G^u < G$).

A decrease in the unloading modulus is due to the continual failure, i.e., the microdefect damage of the material. Ignoring this phenomenon, we obtain the model of a plastic material capable of both hardening and softening. In this case, the stress–strain relation can be written in the form [4]

$$\tau = G\gamma^{\rm e} = G(\gamma - \gamma^{\rm p}), \tag{1.1}$$

where $\gamma^{\rm e}$ and $\gamma^{\rm p}$ are the elastic and plastic shear strains, respectively.

Using (1.1), we obtain $d\tau = G(d\gamma - d\gamma^{\rm p})$. At the same time, the incremental relation $d\tau = G^{\rm p} d\gamma$ is valid. Equating these expressions, we obtain

$$d\gamma^{\rm p} = (1 - G^{\rm p}/G)\, d\gamma. \tag{1.2}$$

Equation (1.2) determines the kinetics of the development of plastic strain in the absence of damages.

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2. Basic and Correcting Problems. We split the initial problem of determining the stress–strain state of the bar loaded by a given torque into basic and correcting problems [4]. The basic problem is the problem of torsion of the elastic bar by a specified torque. Its solution is given by

$$\gamma' = \alpha Mr, \quad \tau' = G\alpha Mr, \quad \psi' = \alpha Ml, \quad \alpha = (0.5\pi GR^4)^{-1},$$

where ψ' is the twist angle produced by the torque M. The correcting problem is the problem of determining the residual stresses in the bar for a given residual plastic strain $\gamma^{\rm p}$ and free boundary. Its solution has the form

$$\eta = \alpha mr, \quad \tau'' = G(\eta - \gamma^{\rm p}), \quad \psi'' = \alpha ml, \quad m = 2\pi \int\limits_0^R G\gamma^{\rm p} r^2\, dr,$$

where ψ'' is the twist angle of the bar with stress-free ends, m is the fictitious torque, η is the shear strain that satisfies the compatibility conditions, and τ'' are the residual stresses.

We introduce the operator P_1 determined by the relation

$$P_1 e(r) = \frac{4r}{R^4} \int\limits_0^R e(r) r^2\, dr,$$

which transforms arbitrary functions $e(r)$ into linear functions. One can see that $\eta = P_1 G\gamma^{\rm p}/G$ and $\tau'' = P_1 G\gamma^{\rm p} - G\gamma^{\rm p}$. Hence, the operator P_1 determines actually the solution of the correcting problem.

For hard loading, the solution of the basic problem has the form

$$\gamma' = \psi r/l, \qquad \tau' = G\psi r/l, \qquad M' = 2\pi G\psi R^4/(4l),$$

where M' is the torque producing the twist angle ψ. In this case, the correcting problem is to determine the residual stresses in the bar with fixed ends. Its solution is given by

$$\eta = 0, \qquad \tau'' = -G\gamma^{\rm p}, \qquad m' = -m,$$

where m' is the torque that holds the bar ends. Obviously, the operator Q_1 determining the solution of the correcting problem is zero and $\tau'' = Q_1 G\gamma^{\rm p} - G\gamma^{\rm p} = -G\gamma^{\rm p}$.

For both soft and hard loadings, the solution of the initial problem for a given $\gamma^{\rm p}$ is a sum of the solutions of the basic and correcting problems.

3. S_t-Criterion for Soft Loading. We give a definition of the stability of equilibrium of the bar in the Lyapunov sense, which is similar to that given in [5].

Definition 1. For soft loading, the state of equilibrium of the bar is stable if, for any $\delta > 0$, there exist parameters $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that, as the torque is increased by a small quantity dM, the inequality $dM < \delta$ yields the inequalities $|d\gamma| < \varepsilon_1$ and $|d\gamma^{\rm p}| < \varepsilon_2$ for each r, where $d\gamma$ and $d\gamma^{\rm p}$ are related to one another by (1.2) and to dM by conditions of equilibrium.

This definition implies that, if the state of equilibrium is disturbed by increasing the torque by dM, the functions $d\gamma$ and $d\gamma^{\rm p}$ must satisfy the initial problem for $d\tau = G(d\gamma - d\gamma^{\rm p})$ and the boundary conditions for dM.

The solutions of the basic and correcting problems yield

$$d\gamma = d\gamma' + d\eta = \alpha r\, dM + G^{-1} P_1 G\, d\gamma^{\rm p}.$$

Substituting the expression for $d\gamma^{\rm p}$ (1.2) into this relation, after simple manipulations, we obtain

$$S_t\, d\gamma = G\alpha r\, dM, \tag{3.1}$$

where

$$S_t = G - P_1 G + P_1 G^{\rm p}. \tag{3.2}$$

Hence, $d\gamma = S_t^{-1} G\alpha r\, dM$. It follows from Definition 1 that the stability of the bar is determined by the properties of the operator S_t, which we call the stability operator. If this operator is reversible, the solution of Eq. (3.1) is unique and the state of equilibrium is stable. If $S_t^{-1} = \infty$ and, hence,

$$S_t = 0, \tag{3.3}$$

the state of equilibrium is unstable.

If the operator P_1 is reversible, we apply the operator P_1^{-1} to equality (3.3) with allowance for (3.2) and infer that the instability criterion (3.3) holds provided the distribution of the tangent shear modulus over the cross section of the bar is given by

$$G^{\rm p} = G - P_1^{-1} G. \tag{3.4}$$

We assume that strain localization does not occur upon the deformation of the bar, i.e., $d\gamma = (r/l)\,d\psi$ in each cross section. Using equality (3.2), we obtain

$$S_t\,d\gamma = \frac{d\psi}{l}\,\frac{4r}{R^4}\int\limits_0^R G^{\rm p}(r) r^3\,dr.$$

It follows that condition (3.3) holds if

$$\int\limits_0^R G^{\rm p} r^3\,dr = 0. \tag{3.5}$$

In this case, a small increase in the torque leads to an unlimited increase in the twist angle.

It is noteworthy that the instability criterion (3.5) coincides with the condition established in [4] under which the iterative process of determining the stress–strain state of the softly loaded bar diverges.

We consider an example. Let the zones of elasticity $V_{\rm e}$ $(0 \leqslant r \leqslant R_{\rm yield})$, hardening V_h $(R_{\rm yield} < r \leqslant R_{\rm t})$, and softening V_s $(R_{\rm t} < r \leqslant R)$ occur in the cross section of the bar for a certain state of equilibrium. We assume that the tangent moduli $G_h^{\rm p}$ and $G_s^{\rm p}$ that refer to the hardening and softening stages, respectively, are constant. We find the value of $G_s^{\rm p}$ for which the state of equilibrium is unstable. From condition (3.5), we obtain

$$\int\limits_0^{R_{\rm yield}} G r^3\,dr + \int\limits_{R_{\rm yield}}^{R_{\rm t}} G_h^{\rm p} r^3\,dr + \int\limits_{R_{\rm t}}^{R} G_s^{\rm p} r^3\,dr = 0.$$

Hence,

$$G_s^{\rm p} = -\frac{G R_{\rm yield}^4 + G_h^{\rm p}(R_{\rm t}^4 - R_{\rm yield}^4)}{R^4 - R_{\rm t}^4}. \tag{3.6}$$

Here $R_{\rm yield}$ and $R_{\rm t}$ are the radii of circles enclosing the elastic and hardening regions, respectively.

We consider an approach based on formula (3.4). It follows from (3.1) that the range of definition and the range of values of the operator S_t are determined by a set Y of linear functions of the form βr $(\beta = \overline{0,\infty})$. We write the operator P_1 in the form of the sum $P_1 = P_1\chi_{\rm e} + P_1\chi_h + P_1\chi_s = P_{1e} + P_{1h} + P_{1s}$. Here

$$\chi_{\rm e} = \begin{cases} 1, & r \in V_{\rm e}, \\ 0, & r \notin V_{\rm e}, \end{cases} \qquad \chi_h = \begin{cases} 1, & r \in V_h, \\ 0, & r \notin V_h, \end{cases} \qquad \chi_s = \begin{cases} 1, & r \in V_s, \\ 0, & r \notin V_s. \end{cases}$$

Thus, the domains of definition of these operators are the sets $\chi_{\rm e}Y$, $\chi_h Y$, and $\chi_s Y$, respectively, and the domain of values is the set Y. Then, we have

$$S_t = G - (P_{1e} + P_{1h} + P_{1s})G + (P_{1e} + P_{1h} + P_{1s})G^{\rm p}. \tag{3.7}$$

Here $P_{1e} = R_{\rm yield}^4/R^4$, $P_{1h} = (R_{\rm t}^4 - R_{\rm yield}^4)/R^4$, and $P_{1s} = (R^4 - R_{\rm t}^4)/R^4$, and the corresponding inverse operators have the form $P_{1e}^{-1} = \chi_{\rm e} R^4/R_{\rm yield}^4$, $P_{1h}^{-1} = \chi_h R^4/(R_{\rm t}^4 - R_{\rm yield}^4)$, and $P_{1s}^{-1} = \chi_s R^4/(R^4 - R_{\rm t}^4)$. To determine the modulus $G_s^{\rm p}$ for which the instability occurs, we equate expression (3.7) to zero and use the operator P_{1s}^{-1}. After some manipulations with allowance for the formulas $\chi_{\rm e} G^{\rm p} = G$, $\chi_h G^{\rm p} = G_h^{\rm p}$, and $\chi_s G^{\rm p} = G_s^{\rm p}$, we obtain the value of $G_s^{\rm p}$ given by formula (3.6).

We now infer whether strain localization is possible in a certain volume of the bar V_a shaped as a cylinder of height a $(0 < a < l)$. We specify the plastic strains $d\gamma_a^{\rm p}(r)$ in the volume V_a and find the solution of the correcting problem. It is given by the equality

$$d\eta = \begin{cases} \alpha m_a r & \text{in } V_a, \\ 0 & \text{in } V_b. \end{cases}$$

Here V_b is the cylinder of height $l - a$ and $m_a = 2\pi \int\limits_0^R G\,d\gamma_a^{\rm p} r^2\,dr$. Then, we obtain

$$P_1G\,d\gamma_a^{\rm p} = P_{1a}G\,d\gamma_a^{\rm p} = \begin{cases} \dfrac{4r}{R^4}\displaystyle\int_0^R G d\gamma_a^{\rm p} r^2\,dr & \text{in } V_a, \\ 0 & \text{in } V_b. \end{cases}$$

In this case, the stability operator is given by

$$S_t\,d\gamma_a = G\,d\gamma_a - P_{1a}G\,d\gamma_a + P_{1a}G_a^{\rm p}\,d\gamma_a = \frac{d\psi_a}{a}\,\frac{4r}{R^4}\int_0^R G_a^{\rm p} r^3\,dr.$$

Here ψ_a is the twist angle of the cylinder V_a, $d\gamma_a = (d\psi_a/a)r$, and $G_a^{\rm p}(r)$ is the distribution of the tangent shear modulus over the cross sections of the cylinder V_a. The instability criterion (3.3) is satisfied if $\int_0^R G_a^{\rm p} r^3\,dr = 0$. Under this condition, the twist angle of the cylinder V_a increases unlimitedly. Consequently, the strain is localized in this volume. A comparison with condition (3.5) shows that the instability of the bar is accompanied by strain localization in a certain volume.

4. S_t-Criterion for Hard Loading. We give a definition of the stability of the state of equilibrium of the bar.

Definition 2. For hard loading, the state of equilibrium of the bar is stable if, for any $\delta > 0$, there exist parameters $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that, as the twist angle increases by a small quantity $d\psi$, the inequality $d\psi < \delta$ yields the inequalities $|d\gamma| < \varepsilon_1$ and $|d\gamma^{\rm p}| < \varepsilon_2$ for each r, where $d\gamma$ and $d\gamma^{\rm p}$ are related to one another by relation (1.2) and to $d\psi$ by conditions of equilibrium.

Let the twist angle increase by $d\psi$. Using the solution of the basic and correcting problems, we infer that

$$d\gamma = d\gamma' + d\eta = (d\psi/l)r + G^{-1}Q_1G\,d\gamma^{\rm p} \tag{4.1}$$

in a state of equilibrium. As expressions (3.1) and (3.2), this equality can be written in the form $S_t\,d\gamma = (d\psi/l)Gr$, where $S_t = G - Q_1G + Q_1G^{\rm p}$. Then, $d\gamma = S_t^{-1}(d\psi/l)Gr$. It follows from Definition 2 that the state of equilibrium becomes unstable if $S_t^{-1} = \infty$ ($S_t = 0$). As in the case of (3.4), we find that the equality $G^{\rm p} = G - Q_1^{-1}G$ must hold at the moment of instability. In the absence of strain localization, we have $Q_1^{-1} = \infty$ ($Q_1 = 0$). Consequently, the instability occurs if $G^{\rm p} = -\infty$.

We determine conditions under which strain localization occurs in a certain volume V_a. We specify the plastic strains $d\gamma_a^{\rm p}(r)$ in this volume and solve the correcting problem. We obtain

$$d\eta = \begin{cases} (1 - a/l)\alpha m_a r & \text{in } V_a, \\ -(a/l)\alpha m_a r & \text{in } V_b. \end{cases}$$

Then, we have

$$Q_1G\,d\gamma_a^{\rm p} = Q_{1a}G\,d\gamma_a^{\rm p} = \begin{cases} (1 - a/l)G\alpha m_a r & \text{in } V_a, \\ -(a/l)G\alpha m_a r & \text{in } V_b. \end{cases}$$

We write equality (4.1) for the region V_a. With allowance for relation (1.2), we obtain

$$d\gamma_a - \Big(1 - \frac{a}{l}\Big)\frac{4r}{GR^4}\int_0^R (G - G_a^{\rm p})d\gamma_a r^2\,dr = \frac{r\,d\psi}{l}.$$

Substituting $d\gamma_a = (d\psi_a/a)r$ into this expression, after simple rearrangement, we find

$$\Big[\frac{a}{l} + \Big(1 - \frac{a}{l}\Big)\frac{4}{R^4G}\int_0^R G_a^{\rm p} r^3\,dr\Big]\frac{d\psi_a}{a} = \frac{d\psi}{l}. \tag{4.2}$$

It follows that the stability operator in the region V_a has the form

$$S_t = \frac{a}{l} + \Big(1 - \frac{a}{l}\Big)\frac{4}{R^4G}\int_0^R G_a^{\rm p} r^3\,dr.$$

Hence, $S_t = 0$ if

$$\int_0^R G_a^{\rm p} r^3\,dr = -\frac{GR^4 a}{4(l-a)}. \tag{4.3}$$

If equality (4.3) is satisfied, the twist angle of the cylinder V_a increases unlimitedly, i.e., the strain is localized in the region V_a. Since the twist angle of the bar is fixed, the twist angle of the cylinder V_b tends to zero, and the cylinder V_b is unloaded.

Obviously, strain localization is accompanied by the loss of stability of the entire bar.

It should be noted that equality (4.3) yields $G^{\rm p} = -\infty$ for $l = a$ (localization is absent). If $l \gg a$, the loading scheme (soft or hard) weakly affects the stability of the twisted bar. Moreover, for $a = 0$, condition (4.3) coincides with the condition of strain localization under soft loading. Since the integral on the left side of formula (4.3) vanishes at the same moment for both schemes of loading, it follows that the instability of the twisted bar made of a plastic material is always related to strain localization in a narrow band of a nearly zero width.

5. *R*-Criterion for Soft Loading. We first consider a material element of unit volume in pure shear and introduce the functional $\rho = d\gamma\, G\, d\gamma - d\gamma^{\rm p}\, G\, d\gamma^{\rm p}$, where $d\gamma^{\rm p}$ is the plastic-strain increment that corresponds to the increment in the total shear strain $d\gamma$. Using the obvious equalities $d\gamma = d\gamma^{\rm e} + d\gamma^{\rm p}$ and $d\tau = G\, d\gamma^{\rm e}$, where $d\gamma^{\rm e}$ is the increment in the elastic shear strain, we obtain the relation $d\tau\, d\gamma + d\tau\, d\gamma^{\rm p} = d\gamma\, G\, d\gamma - d\gamma^{\rm p}\, G\, d\gamma + d\gamma\, G\, d\gamma^{\rm p} - d\gamma^{\rm p}\, G\, d\gamma^{\rm p} = d\gamma\, G\, d\gamma - d\gamma^{\rm p}\, G\, d\gamma^{\rm p} = \rho$. The criterion of physical stability proposed by Drucker [6] implies that the inequalities $\rho > 0$ and $\rho < 0$ are satisfied for stable ($d\tau\, d\gamma > 0$ and $d\tau\, d\gamma^{\rm p} > 0$) and unstable states, respectively.

If $\rho > 0$, then $d\tau(d\gamma + d\gamma^{\rm p}) = d\tau(2d\gamma - d\gamma^{\rm e}) = 2d\tau\, d\gamma - d\gamma^{\rm e}\, G\, d\gamma^{\rm e} > 0$. Since $d\gamma^{\rm e}\, G\, d\gamma^{\rm e} > 0$, then $d\tau\, d\gamma > 0$. If $\rho < 0$, then $d\tau(d\gamma + d\gamma^{\rm p}) = d\tau(d\gamma^{\rm e} + 2d\gamma^{\rm p}) = d\gamma^{\rm e}\, G\, d\gamma^{\rm e} + 2d\tau\, d\gamma^{\rm p} < 0$. Hence, $d\tau\, d\gamma^{\rm p} < 0$.

Thus, the sign of the functional ρ (as well as the Drucker's postulate) determines the physical stability of the material. It is noteworthy that the inequality $\rho > 0$ corresponds to deformation for which the potential elastic-strain energy increases, and the inequality $\rho < 0$ corresponds to deformation for which this energy decreases in this unit volume.

A physically unstable state of the material does not necessarily lead to the loss of stability of the state of equilibrium.

Definition 3. For pure shear, the state of equilibrium of a unit volume is stable if, for any $\delta > 0$ there exists a parameter $\varepsilon > 0$ such that the inequality $d\gamma\, G\, d\gamma < \delta$ yields $d\gamma^{\rm p}\, G\, d\gamma^{\rm p} < \varepsilon$, where $d\gamma$ and $d\gamma^{\rm p}$ are related by (1.2).

Consequently, the instability occurs when an infinitesimal increment in the total strain leads to finite or infinite plastic strains, i.e., $d\gamma^{\rm p}\, G\, d\gamma^{\rm p}/(d\gamma\, G\, d\gamma) = \infty$ or $\rho = -\infty$. Thus, for a unit volume in which the strain produced by hard loading is uniform, this condition holds for $G^{\rm p} = -\infty$. In this case, a so-called unavoidable physical instability occurs in the material.

Disturbing the equilibrium of the bar by increasing the torque by a small quantity dM and summing the functionals ρ over the volume, we obtain

$$R = \int_V \rho\, dV = 2\pi l \int_0^R \rho r\, dr. \tag{5.1}$$

To analyze the stability of the state of equilibrium, we express the quantity $d\gamma^{\rm p}$ in the R-integral (5.1) in terms of $d\gamma$ with the use of equality (1.2) and determine the shear-strain increment $d\gamma$ by the equation of equilibrium

$$dM = 2\pi \int_0^R d\tau\, r^2\, dr = 2\pi \int_0^R G^{\rm p}\, d\gamma\, r^2 dr.$$

With allowance for the equality $d\gamma = (d\psi/l)r$, we obtain

$$R = l(dM)^2 \int_0^R [2GG^{\rm p} - (G^{\rm p})^2] r^3\, dr \Big/ 2\pi G \Big(\int_0^R G^{\rm p} r^3\, dr \Big)^2. \tag{5.2}$$

Drawing the analogy to deformation of a unit volume, we assert that the elastic-strain energy of the bar increases for $R > 0$ and decreases for $R < 0$. The inequality $R < 0$ is the necessary condition of the loss of stability, which occurs for $R = -\infty$. It follows, with allowance for (5.2), that equality (3.5) also determines the moment of instability in this case.

We consider the possibility of strain localization in a certain volume V_a. We write the R-integral in the form

$$R = 2\pi a \int_0^R \rho_a r\,dr + 2\pi(l-a) \int_0^R \rho_b r\,dr, \tag{5.3}$$

where $\rho_a = d\gamma_a\,G\,d\gamma_a - d\gamma_a^{\rm p}\,G\,d\gamma_a^{\rm p}$ and the functional $\rho_b = d\gamma_b\,G\,d\gamma_b - d\gamma_b^{\rm p}\,G\,d\gamma_b^{\rm p}$ is defined in the region V_b. By virtue of equalities (1.2) and $d\gamma_a = (d\psi_a/a)r$ and the equation of equilibrium $dM = 2\pi \int_0^R G_a^{\rm p}\,d\gamma_a r^2\,dr$, the first integral in (5.3) is reduced to a form similar to (5.2), where $G^{\rm p}$ is replaced by $G_a^{\rm p}$. Then, $R = -\infty$ for $\int_0^R G_a^{\rm p} r^3\,dr = 0$. In this case, the loss of stability of the bar is accompanied by strain localization in the volume V_a. It should be noted that this condition coincides with that obtained above.

6. *R*-Criterion for Hard Loading. Let the state of equilibrium of the bar be disturbed by increasing the twist angle by $d\psi$. Using equality (1.2) and equations of equilibrium and expressing all the quantities in terms of $d\psi$, we write the R-integral (5.1) in the form

$$R = \frac{2\pi d\psi^2}{Gl} \int_0^R [2GG^{\rm p} - (G^{\rm p})^2] r^3\,dr.$$

The loss of stability of the bar occurs for $R = -\infty$. In the absence of localization, this condition holds for $G^{\rm p} = -\infty$.

To infer whether strain localization is possible in the region V_a, we write expression (5.3) in the form

$$R = \frac{2\pi d\psi_a^2}{Ga} \int_0^R [2GG_a^{\rm p} - (G_a^{\rm p})^2] r^3\,dr + 2\pi(l-a) \int_0^R \rho_b r\,dr.$$

By virtue of conditions of equilibrium, the quantity $d\psi_a$ should be expressed in terms of $d\psi$ with the use of formula (4.2). As a result, we find that $R = -\infty$ if equality (4.3) is satisfied. Again, we arrive at the result obtained above with the use of the S_t-criterion.

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